Lecture 10 - February 9

Model Checking

Path Satisfaction vs. Model Satisfaction
Unary Temporal Operators: X, G, F

Announcements

- **Lab1** solution coming soon!
- **Lab2** released
- **WrittenTest1** guide & example questions released
  + Verify EECS account on a WSC machine
  + Verify PPY account and Duo Mobile on eClass
- Review session on Monday? 1pm or 2pm?
  ↳ Zoom!

Satisfaction relations

(1) $\pi \models \phi$

(π: path)

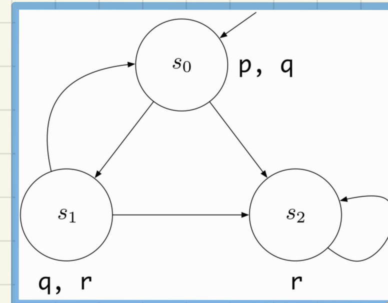
(2) $s, M \models \phi$

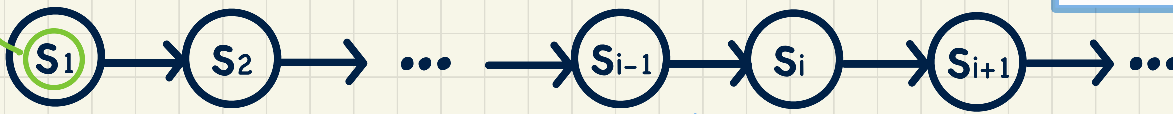
(s: state, M: model)

→ need to consider all π starting from state s.

Path Satisfaction: Logical Operations

A **path** satisfies a **proposition** if its **initial state** ("current state") satisfies it.





e.g. $\pi = s_0 \rightarrow s_2 \rightarrow s_2 \rightarrow \dots$

$\pi \models p$

$\pi \not\models r$

$\pi \models p \iff p \in L(s_1)$ (1st state in path)

$\pi \models \top$ ✓ 1st state satisfies $\top$

$\pi \not\models \bot \iff \neg(\pi \models \bot)$

$\pi \models \neg\phi \iff \neg(\pi \models \phi)$

$\pi \models \phi1 \wedge \phi2 \iff \pi \models \phi_1 \wedge \pi \models \phi_2$

$\pi \models \phi1 \vee \phi2$

$\pi \models \phi1 \Rightarrow \phi2$

$\forall i \cdot i \in \mathbb{N} \wedge i > 0 \Rightarrow \pi^{2i} \models p$

s_2 satisfies p — $\pi^2 \models p$

s_4 satisfies p — $\pi^4 \models p$

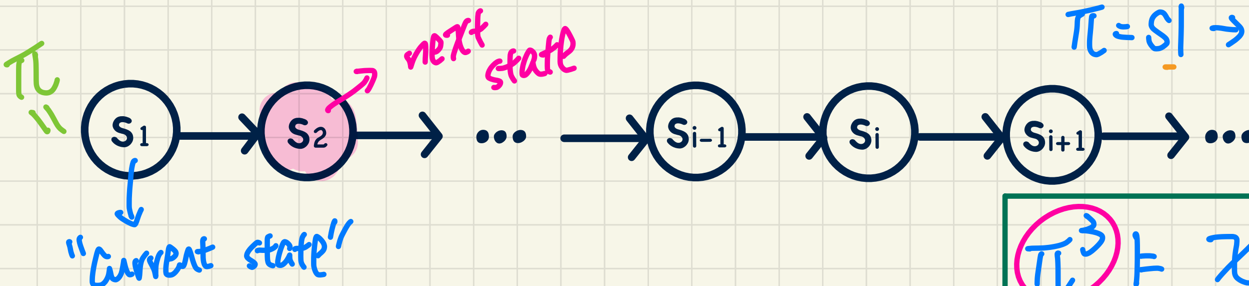
$\forall i \cdot i \in \mathbb{N} \wedge i > 0 \wedge i \% 2 = 0 \Rightarrow \pi^i \models p$

Q: Express that **all** the even-numbered states satisfies a proposition p.

Path Satisfaction: Temporal Operations (1)

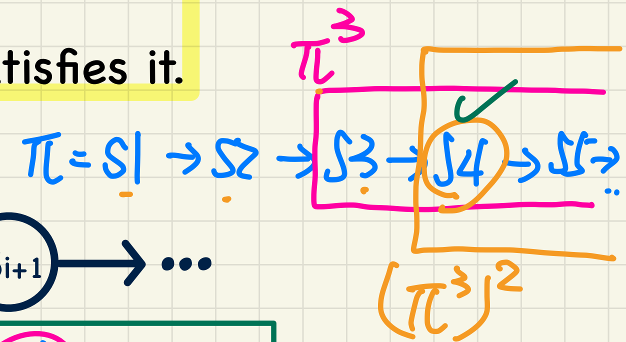
A **path** satisfies $X\phi$

if the **next state** (of the "current state") satisfies it.



Formulation (over a path)

$$\pi \models X\phi \iff \pi^2 \models \phi$$



$$\pi^3 \models X p$$
$$\iff (\pi^3)^2 \models p$$
$$\iff \pi^4 \models p \iff p \in L(S_4)$$

* Q. What is $\pi 3 \models X\ p$ checking?

Model Satisfaction

Given:

- Model $\mathbf{M} = (S, \longrightarrow, L)$
- State $s \in S$
- LTL Formula ϕ

$M, s \vDash \phi$ iff for every path π of M starting at s, $\pi \vDash \phi$.

$s \vDash \phi$

Formulation (over all paths)

model satisfaction

$\pi = s \rightarrow \cdots$ (a valid path w.r.t. M)

$$s \vDash \phi \iff \forall \pi \cdot \pi \text{ starts with } s \Rightarrow \pi \vDash \phi$$

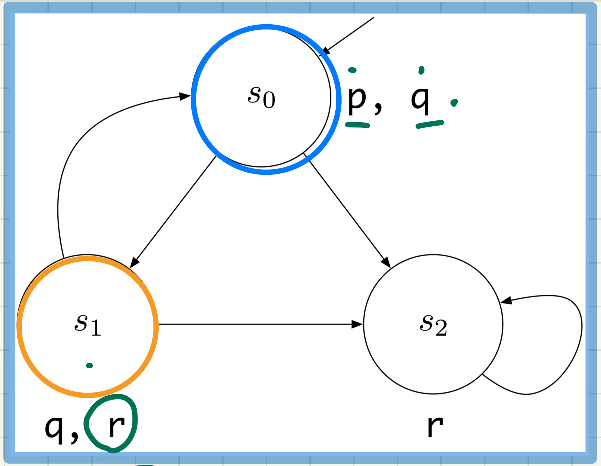
↳ path satisfaction

How to prove vs. disprove $M, s \vDash \phi$?

(1) To prove $s \vDash \phi$, need to show for every possible path π, $\pi \vDash \phi$

(2) To disprove $s \vDash \phi$, provide a witness $\pi = s \rightarrow \cdots$, $\pi \nvDash \phi$.

Model vs. Path Satisfaction: Exercises (1.1)



Recall: $\pi \models p \iff p \in L(s_1)$

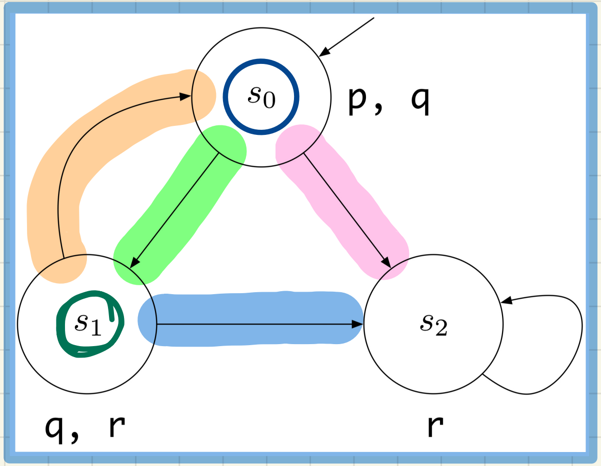
Say: $\pi = s_0 \to s_1 \to s_2 \to s_2 \to \dots$

- $\pi \models \top$ (T)
- $\pi \not\models \bot$ (T)
- $\pi \models p \land q$ (T)
- $\pi \models p \lor q$ (T)
- $\pi \models p \Rightarrow q$ (T)
- $\pi \models r$ (F)
- $\pi \models r \Rightarrow p \land q \land r$ (T)

- $\pi^2 \models p \Rightarrow q$ (T)
- $\pi^2 \models r$ (T)
- $\pi^2 \models r \Rightarrow p \land q \land r$ (F)

Exercise: What if we change the LHS to π^2?

Model vs. Path Satisfaction: Exercises (1.2)



$s \models p \Leftrightarrow$ all π starting at s, $\pi \models p$

$s_0 \models \top$ (T)

$s_0 \not\models \bot$ (T)

$s_0 \models p \wedge q$

$s_0 \models p \vee q$

$s_0 \models p \Rightarrow q$

$s_0 \models r$

$s_0 \models r \Rightarrow p \wedge q \wedge r$

(1) all possible paths starting from s_0 has s_0 as the first state

(2) $\pi \models p \Leftrightarrow p \in L(s_0)$

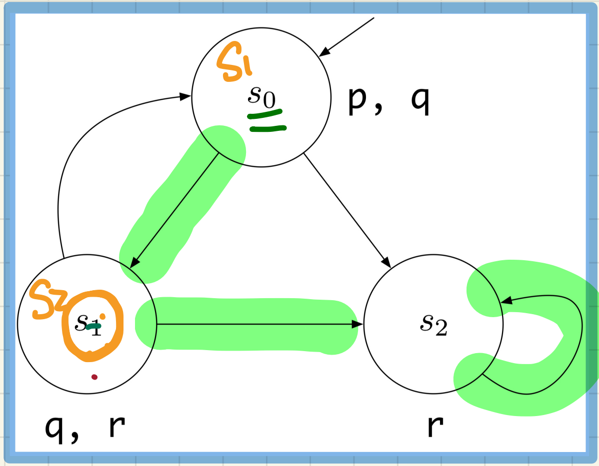
$s_1 \models p \Rightarrow q$ (T) [p: F]

$s_1 \models r$ (T)

$s_1 \models r \Rightarrow p \wedge q \wedge r$ (F) [r: T, p: F]

Exercise: What if we change the LHS to s_1?

Model vs. Path Satisfaction: Exercises (2.1)



Recall: $\pi \models X\,\phi \Leftrightarrow \pi^2 \models \phi$

Say: $\pi = s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow s_2 \rightarrow \ldots$

$\pi \models X\,\top$

$\pi \not\models X\,\bot$

$\pi \models X\,(q \wedge r)$

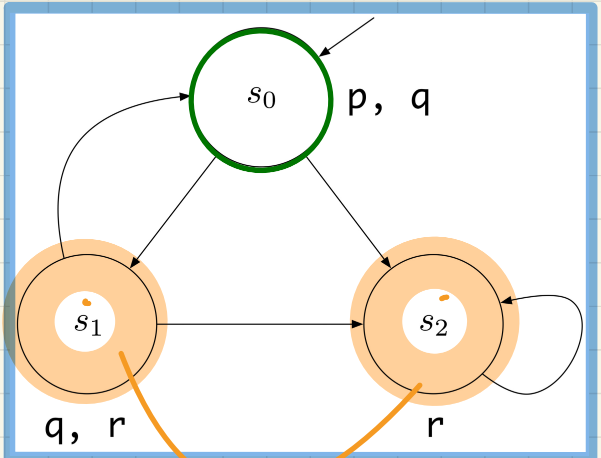
$\pi \models X\,q \wedge r$

$\pi \models X\,(q \Rightarrow r)$

$\pi \models X\,q \Rightarrow r$

Exercise: What if we change the LHS to π^2?

Model vs. Path Satisfaction: Exercises (2.2)



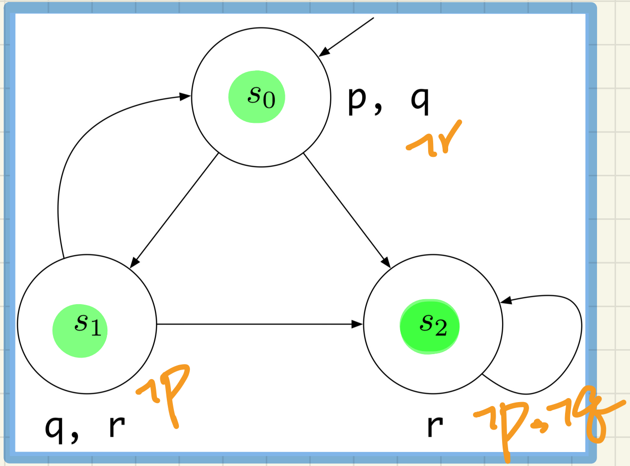
$s \models \phi \Leftrightarrow$ all π starting at s, $\pi \models \phi$

need to consider all paths starting from s_0

- $s_0 \models X\, \top$ — T (possible next states from s_0?)
- $s_0 \not\models X\, \bot$ — T
- $s_0 \models X\,(q \wedge r)$ — F Witness: $s_0 \to s_2 \to \ldots$
- $s_0 \models X\, q \wedge r$ — F Witness: $s_0 \to s_1 \to \ldots$ (s_0 not satisfying r)
- $s_0 \models X\,(q \Rightarrow r)$ — T
- $s_0 \models X\, q \Rightarrow r$ — F Witness: $s_0 \to s_1 \to \ldots$ ($T \Rightarrow F \equiv F$; r is F, q is T) try!

Exercise: What if we change the LHS to s_1?

Model vs. Path Satisfaction: Exercises (3.1)



$\pi \models \mathbf{G}\,\phi \Leftrightarrow \forall i \bullet i \geq 1 \Rightarrow \pi^i \models \phi$

Say: $\pi = s_0 \rightarrow s_1 \rightarrow s_2 \rightarrow s_2 \rightarrow \ldots$

$\pi \models \mathbf{G}\,\top$ (T)

$\pi \not\models \mathbf{G}\,\bot$ (T)

$\pi \models \mathbf{G}\,\neg(p \wedge r)$ → $\neg p \vee \neg r$ (T)

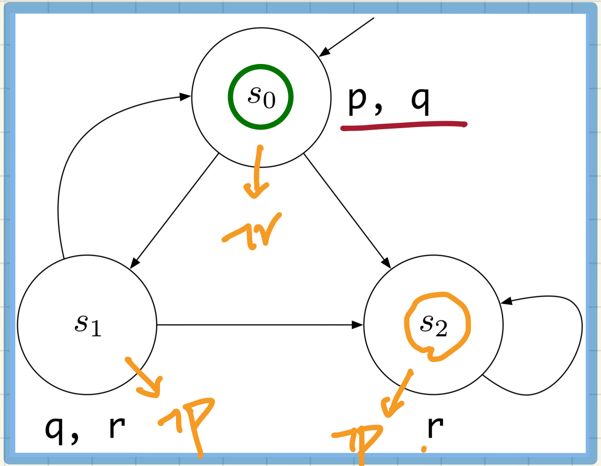
$\pi \models \mathbf{G}\,r$ (F) $s_0 \rightarrow s_1 \rightarrow \cdots$ ($\neg r$ at s_0)

$\pi^2 \models \mathbf{G}\,r$ → $s_1 \rightarrow s_2 \rightarrow s_2 \rightarrow \cdots \models \mathbf{G}\,r$ (T)

To disprove path satisfaction, give a witness state.

Exercise: What if we change the LHS to π^2?

Model vs. Path Satisfaction: Exercises (3.2)



$s \models \phi \Leftrightarrow$ all π starting at s, $\pi \models \phi$

$s_0 \models G\ \top$ ✓ (T)

$s_0 \not\models G\ \bot$ (T)

$s_0 \models G\ \neg(p \wedge r)$ (T) $\equiv \neg p \vee \neg r$ → all paths starting from s_0 over all states

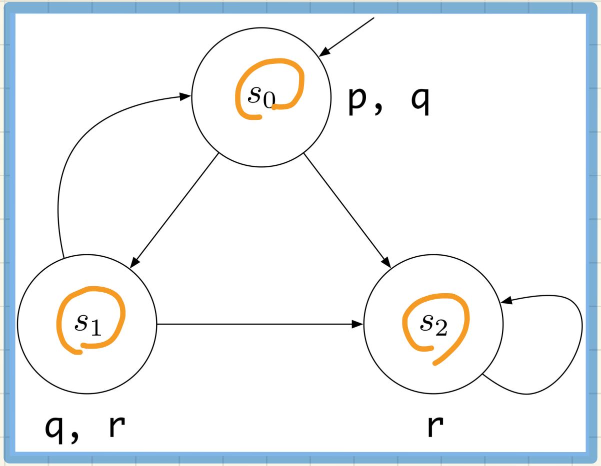
$s_0 \models G\ r$ (F)

witness: $s_0 \rightarrow \dots$ ¬r

$s_2 \models G\ r$ (T)

Exercise: What if we change the LHS to s_1?

Model vs. Path Satisfaction: Exercises (4.1)



$\pi \models \mathbf{F}\,\phi \iff \exists i \bullet i \geq 1 \land \pi^i \models \phi$

Say: $\pi = s_0 \longrightarrow s_1 \longrightarrow s_2 \longrightarrow s_2 \longrightarrow \ldots$

$\pi \models \mathbf{F}\,\top$ T

$\pi \not\models \mathbf{F}\,\bot$ T

$\pi \models \mathbf{F}\,\neg(p \land r)$ T

↳ all states in π actually satisfies $\neg p \lor \neg r$

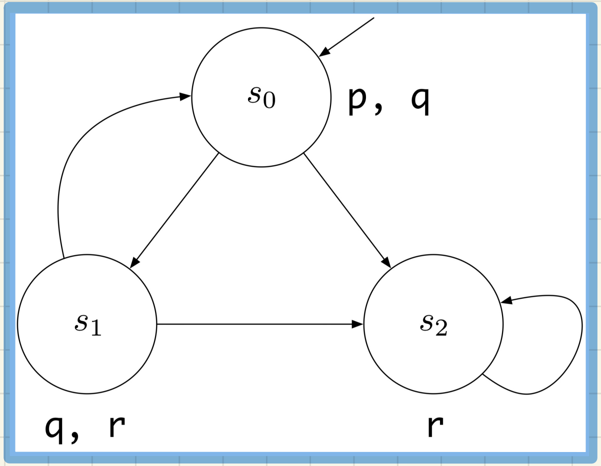
$\pi \models \mathbf{F}\,r$ T

$\pi \models \mathbf{F}\,(q \land r)$

witness: s_1 witness: s_1

Exercise: What if we change the LHS to π^2?

Model vs. Path Satisfaction: Exercises (4.2)



$s \models \phi \Leftrightarrow$ **all π starting at s, $\pi \models \phi$**

$s_0 \models \mathbf{F}\ \top$ (T)

$s_0 \not\models \mathbf{F}\ \bot$ (T)

$s_0 \models \mathbf{F}\ \neg(p \wedge r)$ (T) ∵ every state satisfies $\neg p \vee \neg r$.

$s_0 \models \mathbf{F}\ r$ (T)

$s_0 \models \mathbf{F}\ (q \wedge r)$ F Witness: $s_0 \rightarrow s_2 \rightarrow s_2 \rightarrow \dots$ ($q \wedge r$ never satisfied)

$s \models \mathbf{F}\ \phi$

↳ for each path starting from s ($\pi = s \rightarrow \dots$), there's one state satisfying ϕ.

Exercise: What if we change the LHS to s_1?